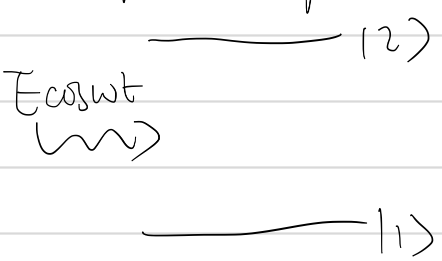
Week 5

From previous lecture



$$|\psi(t)\rangle = a(t)\,e^{-iE_1 t/\hbar}\,|1\rangle + b(t)\,e^{-iE_2 t/\hbar}\,|2\rangle$$

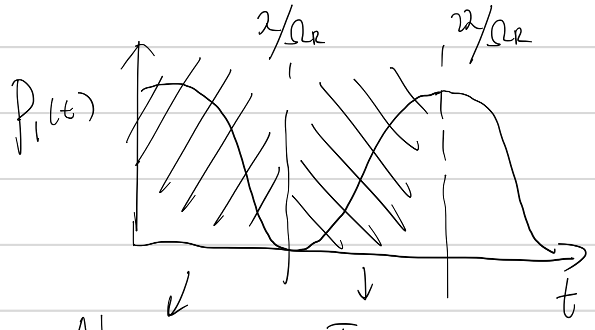
$$a(t) = \cos\frac{\Omega_R t}{2}$$

$$b(t) = i\sin\frac{\Omega_R t}{2}$$

with condition $\hbar\omega = E_2 - E_1 = \hbar\Omega$

and $\Omega_R = \dfrac{\vec{\mu}_{12}\cdot\vec{E}}{\hbar}$: Rabi Frequency.

At this point, we have known how the atom respond to the " EM-wave.



Let's see if we can derive the counterpart for EM-wave.

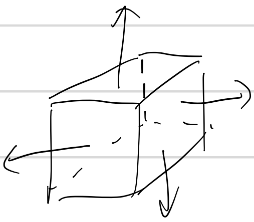
Does EM-wave lose a photon when atom is absorbing?
- - - - - gain a photon - - - - - emission?

This question will be extremely easy to address in full quantum picture, however, classical EM-wave will give us just the same result.

Theorem: Poynting's theorem

The rate of energy transfer (per unit volume) from a region of area equals the rate of work done on a charge distribution plus the energy flux leaving the region.

Why? Energy conservation.



For EM-wave energy, there are two ways it can "escape" the volume: 1° radiation (energy flux) or 2° do work to charge (transfer energy to potential energy in electrical field).

$$\underbrace{\oiint \cdot \vec{S} \cdot d\vec{A}}_{\text{Radiation}} + \underbrace{W}_{\text{work per unit time}} = -\frac{d}{dt} \iiint \left(\frac{\varepsilon_0 |E|^2}{2} + \frac{\mu_0}{2} |H^2| \right) dV$$

An intuitive guess. $\left(\oiint \vec{S} \cdot d\vec{A} \rightarrow \oiint \nabla \cdot \vec{S}\, dV \right)$

Let's work on it.

Something to keep in mind of how to construct the same form on both side of the equation.

Try to see what we need:
We need $\frac{\partial}{\partial t}$; we need $\nabla \cdot \vec{S}$,

Can look in Maxwell equation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \qquad (1)$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial}{\partial t}\vec{E} + \frac{\partial}{\partial t}\vec{P} \qquad (2)$$

Interesting to see: $\vec{H} \cdot \mu_0 \frac{\partial \vec{H}}{\partial t} = \frac{1}{2}\mu_0 \frac{\partial}{\partial t}|\vec{H}|^2$ (Assume H is real)

so we can construct $\frac{\mu_0}{2}|H|^2$ by multiplying $\vec{H}$ to equation (1),
and .... $\frac{\varepsilon_0}{2}|E|^2$ by .... $\vec{E}$ to equation (2).

$$\vec{H} \cdot (\nabla \times \vec{E}) = -\mu_0 \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = -\frac{\mu_0}{2}\frac{\partial}{\partial t}|H|^2$$

$$\vec{E} \cdot (\nabla \times \vec{H}) = \varepsilon_0 \vec{E} \cdot \frac{\partial}{\partial t}\vec{E} + \vec{E} \cdot \partial_t \vec{P} = \frac{\varepsilon_0}{2}\partial_t |\vec{E}|^2 + \vec{E} \cdot \partial_t \vec{P}$$

Then: $\vec{E} \cdot (\nabla \times \vec{H}) - \vec{H} \cdot (\nabla \times \vec{E})$

$$= \frac{\varepsilon_0}{2}\partial_t |\vec{E}|^2 + \frac{\mu_0}{2}\partial_t |H|^2 + \vec{E}\partial_t \vec{P}$$

$\vec{P}$: charge distribution, so this is work.

Now use a little math:

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{A} \times \vec{C}) - \vec{C} \cdot (\vec{A} \times \vec{B})$$

So: $\vec{A} \to \nabla$, $\vec{B} \to \vec{E}$; $\vec{C} \to \vec{H}$

we have:

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot (\nabla \times \vec{H}) - \vec{H} \cdot (\nabla \times \vec{E})$$

$$\therefore \nabla \cdot (\vec{E} \times \vec{H}) = \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0 |E|^2 + \frac{1}{2}\mu_0 |H|^2\right) + \vec{E}\,\partial_t \vec{P}.$$

$\vec{E} \times \vec{H} \equiv \vec{S}$ Poynting vector: energy flux.

$\frac{1}{2}(\varepsilon_0 |E|^2 + \mu_0 |H|^2)$: EM-field energy density

$\vec{E} \cdot \partial_t \vec{P}$: work done to the charge.

For our case, we don't care about energy flux since we are considering standing wave $\vec{E}\cos\omega t$.

Example: atom in resonator (cavity).

$$\nabla \cdot \vec{S} \to 0.$$

So EM-wave energy density change rate:

$$\frac{\partial}{\partial t}\mathcal{E} = -\vec{E}\,\partial_t \vec{P}.$$

For our system, if our calculation is correct, from $t=0 \to t = \frac{\pi}{\Omega_R}$,

$\int_0^{\frac{\pi}{\Omega_R}} \frac{\partial}{\partial t}\mathcal{E}\,dt \to -\hbar\omega$ (absorb one photon to elevate one atom from $|1\rangle$ to $|2\rangle$.

Similarly: $\int_{\frac{\pi}{\Omega_R}}^{\frac{2\pi}{\Omega_R}} \frac{\partial}{\partial t}\mathcal{E}\,dt \to \hbar\omega$: emit one photon.

Now calculate $-\vec{E}\frac{\partial}{\partial t}\vec{P}$

$\vec{P} = Nq\langle\vec{x}\rangle$; $N \to 1$: only one atom per unit volume.

$\vec{P} = q\langle\vec{x}\rangle$

We know: $q\langle\vec{x}\rangle = \langle\psi(t)|\, q\hat{x}\,|\psi(t)\rangle$

$$|\psi(t)\rangle = a(t)e^{-iE_1t/\hbar}|1\rangle + b(t)e^{-iE_2t/\hbar}|2\rangle$$

$\therefore \langle\psi(t)|\, q\hat{x}\,|\psi(x)\rangle$

$$= \left(a^*(t)e^{+iE_1t/\hbar}\langle 1| + b^*(t)e^{+iE_2t/\hbar}\langle 2|\right) qx\cdot\left(a(t)e^{-iE_1t/\hbar} + b(t)e^{-iE_2t/\hbar}\right)$$

We know: $\langle 1|q\hat{x}|1\rangle = \langle 2|q\hat{x}|2\rangle = 0$

$\langle 1|q\hat{x}|2\rangle = \langle 2|q\hat{x}|1\rangle = \vec{\mu}_{12}$.

So we have: $q\langle\vec{x}\rangle$

$$= a^*(t)b(t)e^{-i(E_2-E_1)t/\hbar}\vec{\mu}_{12} + b^*(t)a(t)e^{i(E_2-E_1)t/\hbar}\vec{\mu}_{21}$$

Use: $\hbar\Omega = E_2 - E_1 = \hbar\omega$; $a(t) = \cos\frac{\Omega_R t}{2}$; $b(t) = i\sin\frac{\Omega_R t}{2}$

We have: $q\langle\vec{x}\rangle$

$$= i\vec{\mu}_{12}\cos\frac{\Omega_R t}{2}\sin\frac{\Omega_R t}{2}e^{-i\omega t} - i\vec{\mu}_{12}\cos\frac{\Omega_R t}{2}\sin\frac{\Omega_R t}{2}e^{i\omega t}$$

$$= \vec{\mu}_{12}2\cos\frac{\Omega_R t}{2}\sin\frac{\Omega_R t}{2}\left(\frac{e^{i\omega t} - e^{-i\omega t}}{2i}\right)$$

Use: $2\cos\frac{\Omega_R t}{2}\sin\frac{\Omega_R t}{2} = \sin\Omega_R t$

$$\frac{e^{i\omega t} - e^{-i\omega t}}{2i} = \sin\omega t$$

We have: $\vec{P} = \vec{\mu_{12}} \sin\Omega_R t \sin\omega t$

$$-\vec{E}(t)\partial_t\vec{P} = -\vec{E}\cdot\vec{\mu_{12}}\cos\omega t\cdot(\Omega_R\cos\Omega_R t\sin\omega t + \omega\cos\omega t\sin\Omega_R t)$$

Notice $\Omega_R \ll \omega$

$$\therefore \quad -\vec{E}(t)\partial_t\vec{P} \approx -\omega\cdot\vec{E}\cdot\vec{\mu_{12}}\cdot\cos^2\omega t\sin\Omega_R t$$

$$= -\hbar\omega\cdot\frac{\vec{E}\vec{\mu_{12}}}{\hbar}\cos^2\omega t\sin\Omega_R t = -\hbar\omega\cdot\Omega_R\cdot\cos^2\omega t\sin\Omega_R t$$

When we do integral from $t=0$ to $t=\frac{\pi}{\Omega_R}$, the time scale is much longer than $\frac{1}{\omega}$, so we can take average of $\cos^2\omega t$

Easy to see it's $\frac{1}{2}$

$$\frac{1}{NT}\int_0^{NT}\cos^2\omega t\,dt = \frac{1}{NT}\int_0^{NT}\frac{1}{2}(1+\underbrace{\cos 2\omega t}_{\to 0})\,dt = \frac{NT}{2NT} = \frac{1}{2}.$$

$$\therefore \quad -\vec{E}(t)\partial_t\vec{P} = -\frac{\hbar\omega\Omega_R}{2}\sin\Omega_R t$$

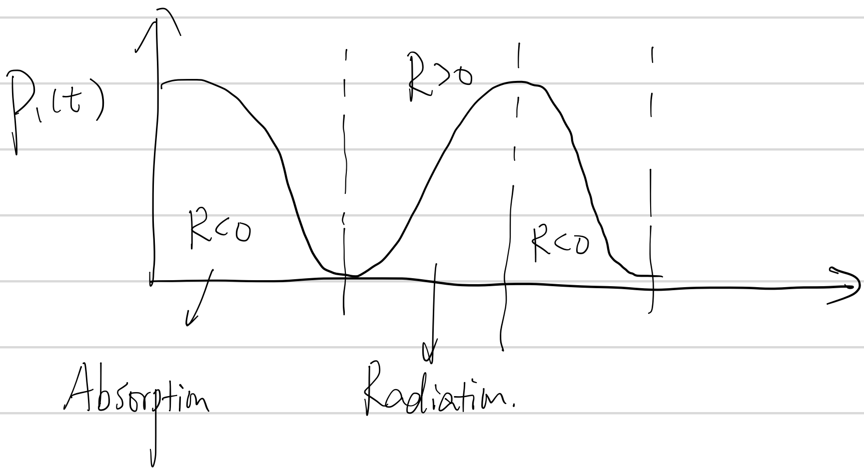
So energy change is:

$$\delta E = -\int_0^{\frac{\pi}{\Omega_R}} \frac{\hbar\omega}{2}\cdot \Omega_R \sin\Omega_R t \, dt = -\int_0^{\pi} \frac{\hbar\omega}{2} \sin\Omega_R t \cdot d(\Omega_R t)$$

$$= -\frac{\hbar\omega}{2}\left(-\cos\Omega_R t \Big|_{\Omega_R t=0}^{\Omega_R t=\pi}\right) = -\hbar\omega$$

The EM-field lose energy of $\hbar\omega$, one photon.

We can actually see: $-\vec{E}\,\partial_t\vec{P} = -\frac{\hbar\omega\Omega_R}{2}\sin\Omega_R t = R$



Let's build a better link between atomic scale solution and macroscopic scale optical property. χ, or ε_r.

A critical difference between a single atom and a large collection of atom is the phase.

Naturally, atoms will not all be in ground state at $t=0$.

Even if they are: atom A could be in $|\psi_A\rangle = e^{-iE_i t/\hbar + i\phi_A}|1\rangle$

atom B could be $|\psi_B\rangle = e^{-E_i t/\hbar + i\phi_B}|1\rangle$.

These two are the main sources of decoherence

You have a quantum state: $|\psi\rangle = a|1\rangle + b|2\rangle$

Decohere gives you: $|\psi\rangle \rightarrow a'|1\rangle + b'|2\rangle$

$\left|\frac{a'}{a}\right| \neq 1$, amplitude change

$\frac{a'}{a} = \left|\frac{a'}{a}\right| e^{i\Delta\phi}$: phase change

So how to calculate $\vec{P}$ so to get χ, ε_r?

$$\vec{P}(t) = N \langle \psi(t) | q\vec{x} | \psi(t) \rangle \equiv N \langle \vec{\mu} \rangle$$

$$\langle \vec{\mu} \rangle = \langle \psi(t) | q\hat{x} | \psi(t) \rangle = a^*(t) b(t) e^{-i\Omega t} \vec{\mu}_{12} + b^*(t) a(t) e^{i\Omega t} \vec{\mu}_{21}$$

We need to consider decoherence to get $a^* b$ and $b^* a$

$$\vec{P}(t) = \vec{N} \{ \vec{\mu}_{21} b^*(t) a(t) e^{i\Omega t} + \vec{\mu}_{12} a^*(t) b(t) e^{-i\Omega t} \}$$

As $\vec{E} = \vec{E} \cos\omega t = \dfrac{\vec{E} e^{-i\omega t} + \vec{E} e^{+i\omega t}}{2}$

We could have:

$$\vec{P} = \frac{\varepsilon_0 \chi(\omega) E}{2} e^{i\omega t} + \frac{\varepsilon_0 \chi(-\omega)}{2} e^{-i\omega t}$$

Need to solve $a^*(t) b(t)$ and $b^*(t) a(t)$.

Define: $p_{12} = a^*(t) b(t) e^{-i\Omega t}$; $p_{21} = b^*(t) a(t) e^{i\Omega t}$

$$\vec{P} = (\vec{\mu}_{12} p_{12} + \vec{\mu}_{21} p_{21}) N.$$

Construct equation for p_{12}, p_{21}

Now we can't assume $\omega = \Omega$, that would be too limited

$$\rho_{12} = a^*(t)b(t)e^{-i\Omega t} \qquad \rho_{21} = b^*(t)a(t)e^{i\Omega t}$$

When there is no $\vec{E}$ field, $a(t)$, $b(t)$ will be constant, so we have:

$$\dot{\rho}_{12} = -i\Omega\rho_{12}; \quad \dot{\rho}_{21} = i\Omega\rho_{21}$$

However, there is also decoherence that kills the dipole. The simplest form would be exponential decay of dipole:

$$\dot{\rho}_{12} = -i\Omega\rho_{12} - \frac{1}{T_2}\rho_{12}; \quad \dot{\rho}_{21} = -i\Omega\rho_{21} - \frac{1}{T_2}\rho_{21}$$

This is the equation when you have no $\vec{E}$ field.

Now add in the effect of $\vec{E}$ field:

We know $\vec{E}$ field can drive $a^*(t)$ and $b(t)$, and we have previously solved something.

So: $$\dot{\rho}_{12} = \dot{a}^*(t)b(t)e^{-i\Omega t} + a^*(t)\dot{b}(t)e^{-i\Omega t} - i\Omega\rho_{12} - \frac{1}{T_2}\rho_{12}$$

Now we sustitute in our previous equation

$$\begin{cases} \dot{a}(t) = \dfrac{i\vec{\mu}_{12}\cdot\vec{E}}{2\hbar} e^{i(\omega-\Omega)t} b(t) \\ \dot{b}(t) = \dfrac{i\vec{\mu}_{21}\cdot\vec{E}}{2\hbar} e^{-i(\omega-\Omega)t} a(t) \end{cases}$$

Note: the reason we can do this is because we have assume the coherent drive of $\vec{E}$ and other effect are independent. So in a rate equation, you write:

$$\frac{d}{dt} = \text{Effect 1} + \text{Effect 2} + \ldots\ldots$$

So $\dot{a}^*(t)\,b(t)$, and $a^*(t)\dot{b}(t)$ is non-zero, the contribution is from $\vec{E}$ field.

Use $$\dot{a}^*(t) = -\frac{i\vec{\mu}_{12}\cdot\vec{E}}{2\hbar}\, e^{i(\omega-\Omega)t}\, b^*(t)$$

$$\dot{b}(t) = \frac{i\vec{\mu}_{21}\cdot\vec{E}}{2\hbar}\, e^{-i(\omega-\Omega)t}\, a(t)$$

We have: $$\dot{\rho}_{12} = -\frac{i\vec{\mu}_{12}\cdot\vec{E}}{2\hbar}|b(t)|^2 e^{-i\omega t} + \frac{i\vec{\mu}_{21}\cdot\vec{E}}{2\hbar}|a|^2 e^{-i\omega t} - i\Omega\rho_{12} - \frac{\rho_{12}}{T_2}$$

$$= -\frac{i\vec{\mu}_{12}\cdot\vec{E}}{2\hbar}\left(|b|^2-|a|^2\right) e^{-i\omega t} - i\Omega\rho_{12} - \frac{\rho_{12}}{T_2}$$

Similarly: $\dot{\rho}_{21} = \ldots\ldots$, but we don't need to solve it.
Just focus on $\dot{\rho}_{12}$

Something to keep in mind first: $(|b|^2-|a|^2)N = N_2 - N_1 = \Delta N$

$\Delta N > 0$: Population inversion

This calculation should give us loss and gain.

We need to solve $\rho_{12}(t)$ from

Let $\rho_1 = |a|^2$; $\rho_2 = |b|^2$

$$\partial_t \rho_{12} = \left(-i\Omega - \frac{1}{T_2}\right)\rho_{12}(t) - \frac{i\vec{\mu}_{12}\cdot\vec{E}}{2\hbar}(\rho_2(t) - \rho_1(t))e^{-i\omega t}$$

Math tip:

initial condition

$$\rho_{12}(t) = \rho_{12}(0)e^{-(i\Omega+\frac{1}{T_2})t} - \frac{i\vec{\mu}_{12}}{2\hbar}\cdot\int_0^t dt' e^{-(i\Omega+\frac{1}{T_2})(t-t')}\cdot\vec{E}(t')e^{-i\omega t'}(\rho_2(t') - \rho_1(t'))$$

And for $t \gg T_2$, the first term vanish ($e^{-\frac{t}{T_2}} \ll 0$).

Let's verify this:

For $\int_0^x f(x,x')dx'$, its differential is

$$\frac{d}{dx}\int_0^x f(x,x')\,dx = \int_0^x \frac{d}{dx}f(x,x')\cdot dx' + f(x, x'=x).$$

So: $$\rho_{12}(t) = -\frac{i\vec{\mu}_{12}}{2\hbar}\int_0^t dt' e^{-(i\Omega+\frac{1}{T_2})(t-t')}\vec{E}(t')e^{-i\omega t'}\{\rho_2(t') - \rho_1(t')\}$$

$$\dot{\rho}_{12}(t) = -\left(i\Omega + \frac{1}{T_2}\right)\cdot \underbrace{-\frac{i\vec{\mu}_{12}}{2\hbar}\int_0^t dt' e^{-(i\Omega+\frac{1}{T_2})(t-t')}\vec{E}(t')e^{-i\omega t'}[\rho_2(t') - \rho_1(t')]}_{\to\ \rho_{12}(t)}$$

$$- \frac{i\vec{\mu}_{12}\cdot\vec{E}(t)}{2\hbar}e^{-i\omega t}[\rho_2(t) - \rho_1(t)]$$

$$= -\left(i\Omega + \frac{1}{T_2}\right)\rho_{12}(t) - \frac{i\vec{\mu}_{12}\cdot\vec{E}(t)e^{-i\omega t}}{2\hbar}[\rho_2(t) - \rho_1(t)]$$

Correct. Let's now calculate this integral:

$$\rho_{12}(t) = \frac{-i\vec{\mu}_{12}}{2\hbar}\int_0^t dt' \, e^{-(i\Omega+\frac{1}{T_2})(t-t')}\vec{E}(t')e^{-i\omega t'}[\rho_2(t')-\rho_1(t')]$$

Because of $\frac{1}{T_2}$, the main contribution comes from $\frac{(t-t')}{T_2} \sim 1$

And since we assumed $\frac{t}{T_2} \gg 1$, so $\int_0^t \to \int_{-\infty}^t$

Next, apply week coupling condition:

$\Omega_R = \frac{\vec{\mu}\cdot\vec{E}}{\hbar} \ll \frac{1}{T_2}$: the speed of converting atom from $|1\rangle \to |2\rangle$ is much much slower than the decohent speed of the system

This is true for most classical system. (room temperature, thermal state)

What this means is that within the time scale T_2, $\vec{E}(t')$, $\rho_2(t')$, $\rho_1(t')$ does not change much.
(to change them, one need $\frac{1}{\Omega_R}$ time)

So we can simply use $\vec{E}(t)$ to replace $\vec{E}(t')$ because from $t-t' \sim t-T_2$ to t', $\vec{E}$ does not change much.

$\vec{E}(t') \to \vec{E}(t)$; $\rho_2(t') \to \rho_2(t)$; $\rho_1(t') \to \rho_1(t)$.

$$\rho_{12}(t) = -\frac{i\vec{\mu}_{12}\vec{E}(t)}{2\hbar}e^{-(i\Omega+\frac{1}{T_2})t}[\rho_2(t)-\rho_1(t)]\int_{-\infty}^{t} e^{i(\Omega-\omega)t'}\cdot e^{\frac{1}{T_2}t'}dt'$$

This is Rate equation Approximation.

calculate:

$$\int_{-\infty}^{t} e^{[i(\Omega-\omega)+\frac{1}{T_2}]t'} dt'$$

$$\int_{-\infty}^{a} e^{\alpha x} \cdot dx = \frac{e^{\alpha x}}{\alpha}\Big|_{-\infty}^{a} = \frac{1}{\alpha}\{e^{\alpha a} - e^{\alpha\cdot(-\infty)}\}$$

$$\therefore = \frac{1}{i(\Omega-\omega)+\frac{1}{T_2}}\left[e^{(i(\Omega-\omega)+\frac{1}{T_2})t} - e^{\{i(\Omega-\omega)+\frac{1}{T_2}\}\cdot(-\infty)}\right]$$

$\to 0.$

$$= \frac{e^{i(\Omega-\omega)+\frac{1}{T_2})t}}{i(\Omega-\omega)+\frac{1}{T_2}}$$

In combine: $$\rho_{12}(t) = -\frac{i\vec{\mu}_{12}\cdot\vec{E}}{2\hbar}\,[\rho_2(t)-\rho_1(t)]\,\frac{e^{(-i\Omega-\frac{1}{T_2})t}\,e^{[i(\Omega-\omega)+\frac{1}{T_2}]t}}{[i(\Omega-\omega)+\frac{1}{T_2}]}$$

$$= -\frac{i\vec{\mu}_{12}\cdot\vec{E}\,[\rho_2(t)-\rho_1(t)]\,e^{-i\omega t}}{2\hbar(i(\Omega-\omega)+1/T_2)}$$

$$\therefore \vec{P} = \vec{\mu}_{12}\cdot N\cdot\vec{\rho}_{12} + c.c.$$

$$= \frac{-i\vec{\mu}_{12}\cdot\vec{\mu}_{12}(N_2-N_1)}{2\hbar[i(\Omega-\omega)+\frac{1}{T_2}]}\cdot\vec{E}e^{-i\omega t} + c.c.$$

And $$\vec{P} = \frac{\varepsilon_0\chi(-\omega)\vec{E}e^{-i\omega t}}{2} + \frac{\varepsilon_0\chi(\omega)\vec{E}e^{i\omega t}}{2}$$

We have $$\boxed{\chi(-\omega) = \frac{-i}{\varepsilon_0\hbar}\left[\frac{N_2-N_1}{i(\Omega-\omega)+\frac{1}{T_2}}\right]\underbrace{\vec{\mu}_{12}\vec{\mu}_{21}^{T}}_{\text{Tensor.}}}$$

Later we will examine gain and loss, refractive index from this result.

A shortcut to get $\rho_{12}(t)$

$$\partial_t \rho_{12} = \left(-i\Omega - \frac{1}{T_2}\right)\rho_{12}(t) - \frac{i\vec{\mu}_{12}\cdot\vec{E}(t)}{2\hbar}\left(\rho_2(t) - \rho_1(t)\right)e^{-i\omega t}$$

Let $\rho_{12}(t) = \tilde{\rho}_{12}(t)\, e^{-i\omega t} + C_0$

Because of damping term, any initial condition $C_0 \to 0$ at $t \gg 0$.

$$\rho_{12}(t) = \tilde{\rho}_{12}(t)\, e^{-i\omega t}$$

$\boxed{\text{Rate equation approximation}}$

Assume $\vec{E}(t), \rho_2(t), \rho_1(t)$ varies much slower than $-\frac{1}{T_2}$ term.

So: $\frac{1}{T_2}\tilde{\rho}_{12}(t) \gg \dot{\tilde{\rho}}_{12}(t)$.

Then we can treat $\tilde{\rho}_{12}(t)$ as steady state. $\dot{\tilde{\rho}}_{12}(t) \to 0$.

$$\Rightarrow \quad -i\omega\, \tilde{\rho}_{12}(t)\, e^{-i\omega t} = \left(-i\Omega - \frac{1}{T_2}\right)\tilde{\rho}_{12}(t)\, e^{-i\omega t} - \frac{i\vec{\mu}_{12}\cdot\vec{E}(t)}{2\hbar}\left[\rho_2(t) - \rho_1(t)\right]e^{-i\omega t}$$

$$\Rightarrow \quad \tilde{\rho}_{12}(t)\, e^{-i\omega t} = \rho_{12}(t) = \frac{-i\vec{\mu}_{12}\cdot\vec{E}(t)\left[\rho_2(t) - \rho_1(t)\right]e^{-i\omega t}}{i(\Omega - \omega) + \frac{1}{T_2}}$$

Same result.